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Spin structure function of the virtual photon *

Ken Sasaki[a] and Tsuneo Uematsu[b]

[a]Department of Physics, Faculty of Engineering, Yokohama National University, Yokohama 240-8501, Japan

[b]Department of Fundamental Sciences, FIHS, Kyoto University, Kyoto 606-8501, Japan

We investigate the spin structure of the virtual photon beyond the leading order in QCD. The first moment of the virtual photon spin structure function $g_1^\gamma(x, Q^2, P^2)$ with QCD effects turns out to be non-vanishing in contrast to the real photon case. Numerical analysis for virtual as well as real photon case is presented.

1. INTRODUCTION

In the last several years the nucleon's spin structure functions have been extensively studied by deep inelastic scattering of polarized leptons on polarized nucleon targets. Recently there has been growing interest in the polarized photon structure function. Especially its first moment has attracted much attention in the literature in connection with the axial anomaly. Now the information on the spin structure of the photon would be obtained by the resolved photon processes in the polarized electron and proton collision in the polarized version of HERA. More directly the spin-dependent structure function of photon g_1^γ can be measured by the polarized e^+e^- collisions in the future linear colliders (Fig.1).

We investigate the polarized virtual photon structure function $g_1^\gamma(x, Q^2, P^2)$ beyond the leading order in QCD, in the kinematical region:

$$\Lambda^2 \ll P^2 \ll Q^2 \tag{1}$$

where $-Q^2(-P^2)$ is the mass squared of the probe (target) photon (Fig.1) with Λ being the QCD scale parameter. The next-to-leading order (NLO) analysis is now possible since the required spin-dependent two-loop splitting functions in DGLAP evolution equation or equivalently the two-loop anomalous dimensions of the relevant operators have been recently calculated [1, 2]. The advantage in studying the virtual photon target is that we can calculate the whole structure function by the perturbative method, in contrast to the case of the real photon target, where there exist non-perturbative pieces in NLO. Our motivation here is to carry out the analysis of the polarized structure function at the same level as in the unpolarized case [3].

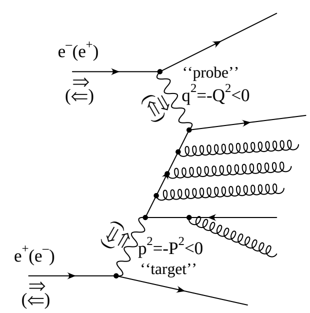

Figure 1. Deep inelastic scattering on a virtual photon in polarized e^+e^- collision.

2. QCD CALCULATION of g_1^γ

The same framework used in the analysis of nucleon spin structure functions can be applied to the present case. Namely we can base our argument either on the operator product expansion (OPE) supplemented by the renormalization group (RG), or on the DGLAP type parton evo-

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lution equation. The n-th moment of g_1^γ for the kinematical region (1) is given by [4]

$$\int_0^1 dx x^{n-1} g_1^\gamma(x,Q^2,P^2) = \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\times$$
$$\Big[\sum_{i=+,-,NS} L_i^n \frac{4\pi}{\alpha_s(Q^2)}\Big\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\lambda_i^n/2\beta_0+1}\Big\}$$
$$+\sum_{i=+,-,NS} \mathcal{A}_i^n \Big\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\lambda_i^n/2\beta_0}\Big\}$$
$$+\sum_{i=+,-,NS} \mathcal{B}_i^n \Big\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\lambda_i^n/2\beta_0+1}\Big\}$$
$$+ \quad \mathcal{C}^n + \quad \mathcal{O}(\alpha_s) \quad \Big] \tag{2}$$

where L_i^n, $\mathcal{A}_i^n$, $\mathcal{B}_i^n$ and $\mathcal{C}^n$ are computed from the one- and two-loop anomalous dimensions together with one-loop coefficient functions. All of them are shown to be renormalization-scheme independent. $\alpha_s(Q^2)$ is the QCD running coupling constant, and λ_i^n $(i=+,-,NS)$ denote the eigenvalues of one-loop anomalous dimension matrix.

3. SUM RULE FOR g_1^γ

For a real photon ($P^2=0$), the 1st moment vanishes to all orders of $\alpha_s(Q^2)$ in QCD [5]:

$$\int_0^1 dx g_1^\gamma(x,Q^2) = 0. \tag{3}$$

Now the question is what about the $n=1$ moment of the virtual photon case. Taking $n\to 1$ limit of (2) the first 3 terms vanish as

$$L_i^n \to 0, \sum_i \mathcal{A}_i^n\{\ \} \to 0, \sum_i \mathcal{B}_i^n\{\ \} \to 0. \tag{4}$$

Denoting $\langle e^4\rangle = \sum_{i=1}^{n_f} e_i^4/n_f$ (e_i: the i-th quark charge, n_f: the number of flavors), we have

$$\mathcal{C}^{n=1} = 12\beta_0\langle e^4\rangle (B_G^n + A_{qG}^n)|_{n=1}. \tag{5}$$

Here we note that the sum of the one-loop coefficient function B_G^n and the finite photon matrix element of quark operator A_{qG}^n is renormalization-scheme independent and equal to $-2n_f$ for $n=1$.

Therefore we have

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2) = -\frac{3\alpha}{\pi}\sum_{i=1}^{n_f} {e_i}^4 + \mathcal{O}(\alpha_s) \tag{6}$$

We can go a step further to $\mathcal{O}(\alpha_s)$ QCD corrections which turn out to be [4]

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2)$$
$$= -\frac{3\alpha}{\pi}\left[\sum_{i=1}^{n_f} e_i^4\left(1-\frac{\alpha_s(Q^2)}{\pi}\right) - \frac{2}{\beta_0}(\sum_{i=1}^{n_f} e_i^2)^2\left(\frac{\alpha_s(P^2)}{\pi}-\frac{\alpha_s(Q^2)}{\pi}\right)\right]$$
$$+\mathcal{O}(\alpha_s^2). \tag{7}$$

This result coincides with the one obtained by Narison, Shore and Veneziano in ref.[6], apart from the overall sign for the definition of g_1^γ.

4. NUMERICAL ANALYSIS

The polarized structure function $g_1^\gamma(x,Q^2,P^2)$ as a function of x is obtained by the inverse Mellin transform of the moments (2). In Fig.2, we have plotted the result for $n_f=3$, $Q^2=30\mathrm{GeV}^2$, $P^2=1\mathrm{GeV}^2$ with $\Lambda=0.2\mathrm{GeV}$. Here we have shown the Box (tree) diagram contribution:

$$g_1^{\gamma(\mathrm{Box})}(x,Q^2,P^2) = (2x-1)\frac{3\alpha}{\pi}n_f\langle e^4\rangle \ln\frac{Q^2}{P^2} \tag{8}$$

and the Box diagram contribution including non-leading correction (Box, NL), the leading order (LO) QCD correction and the next-to-leading order (NLO) QCD correction. In this analysis we observe that i) The NLO QCD correction is significant at large x as well as at low x. ii) No sizable change for the normalized structure function is seen for different values of Q^2 and P^2. iii) The $n_f=4$ case has been examined as well, but the normalized structure function is insensitive to the number of active flavors, n_f.

5. CONCLUDING REMARKS

In this talk we have discussed the virtual photon's spin structure function $g_1^\gamma(x,Q^2,P^2)$ for the kinematical region (1), to the NLO in QCD. The result we obtained is independent of renormalization scheme. The first moment of g_1^γ is non-vanishing in contrast to the real photon case, where we have vanishing sum rule which is an extension of Drell-Hearn-Gerasimov sum rule. The

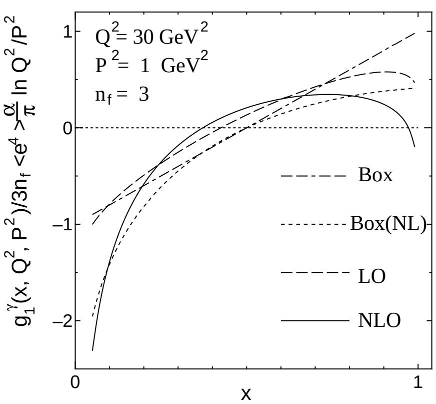

Figure 2. Spin structure function, $g_1^\gamma(x,Q^2,P^2)$ in units of $(3\alpha n_f\langle e^4\rangle/\pi)\ln(Q^2/P^2)$, for $Q^2 = 30\mathrm{GeV}^2$, $P^2 = 1\mathrm{GeV}^2$, $n_f = 3$ and $\Lambda = 0.2$ GeV.

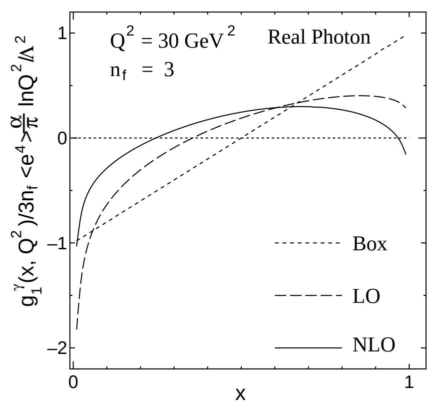

Figure 3. Real photon spin structure function $g_1^\gamma(x,Q^2)$ in units of $(3\alpha n_f\langle e^4\rangle/\pi)\ln(Q^2/\Lambda^2)$, for $Q^2 = 30\mathrm{GeV}^2$, $n_f = 3$ and $\Lambda = 0.2$ GeV.

NLO QCD corrections are significant at large x as well as at low x. The real photon case ($P^2 = 0$) consists of the perturbative piece and non-perturbative piece, as

$$g_1^\gamma(x,Q^2) = g_1^\gamma(x,Q^2)|_{\text{pert.}} + g_1^\gamma(x,Q^2)|_{\text{nonpert.}} \quad (9)$$

The perturbative parts can formally be recovered by setting $P^2 = \Lambda^2$ in (2). In Fig.3 we have plotted the point-like piece of $g_1^\gamma(x,Q^2)$ for $Q^2 = 30\mathrm{GeV}^2$. The LO results coincides with result by Sasaki in ref.[7], in which the structure function $W_4^\gamma = g_1^\gamma/2$ was used. The NLO results are qualitatively consistent with the analysis by Stratmann and Vogelsang [8]. The future subjects to be pursued are the following; i) We have to understand how the transition occurs from the vanishing first moment for real photon ($P^2 = 0$) to non-vanishing one for virtual photon ($P^2 \gg \Lambda^2$). ii) We should study the spin-dependent parton distribution functions inside the polarized virtual photon. The preliminary result shows that the NLO effects are significant at small x and also at large x in the $\overline{\mathrm{MS}}$ scheme [9]. iii) It would be intriguing to investigate another structure function $g_2^\gamma(x,Q^2,P^2)$ for which twist-2 and twist-3 operators contribute. iv) The power corrections of the form $(P^2/Q^2)^k (k = 1,2,\cdots)$ arising from target mass effects and higher-twist effects should be explored. v) An extension to time-like polarized virtual photon in e^+e^- process is now under study.

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